ALGEBRAIC AND TOPOLOGICAL QUANTUM FIELD THEORY
FALL 2015

Prerequisites: Linear algebra, real analysis and basic functional analysis. Some knowledge about quantum mechanics, special relativity and category theory might be useful, but is not necessary.

The goal of this course is to give an introduction to Algebraic Quantum Field Theory and Topological Quantum Field Theory.

Quantum field theory (QFT) was invented to describe particle physics at high energies, but also gives rise to interesting mathematics. The plan of the lecture is to start with a mild introduction to quantum theory and symmetries. I will discuss the axiomatization of quantum field theory due to Haag–Kastler. In this approach we will study free field examples and relation to the usual definition of quantum fields as operator valued distributions and discuss the theory of superselection sectors, which gives rise to tensor categories.

In the second part, I will give an introduction to topological quantum field theory (TQFT), which is of a bit different flavor. TQFTs are used in mathematics to calculate topological invariants of spaces, while in physics it describes low energy effective theories in condensed matter physics. We will focus on how TQFTs are obtained from tensor categories.

Finally, we will be studying conformal field theory (CFT) in low dimensions from an algebraic point of view. Here, in contrast to higher dimensions, many interesting and non-trivial models can be rigorously constructed. We will encounter subfactors and see that so-called rational CFTs give rise to TQFTs and topological invariants, like the Jones polynomial. If time permits we will talk about classification results in rational CFT.