Problem: Standard pairs and 2D Wigner representations

Let (H, T) be the unique irreducible standard pair on a Hilbert space $\mathcal{H}$, i.e. $H \subset \mathcal{H}$ is a standard space and $T(t) = \mathrm{e}^{\mathrm{i}tP}$ with $P > 0$ and $T(t)H \subset H$ for $t \geq 0$. Irreducible means that (H, T) cannot be written as a direct sum of two such pairs. Let us define a new one-parameter group $V_m(s) = \mathrm{e}^{-\mathrm{i}m^2 s/P}$, then (as seen in the lecture) $V_m(s)$ commutes with $T(t)$ and $V_m(s)H \subset H$ for $s \geq 0$.
For (x, λ) the Poincare transformation

$$y \mapsto \begin{pmatrix} \cosh\lambda & \sinh\lambda \\ \sinh\lambda & \cosh\lambda \end{pmatrix} y + x$$

let us define a unitary

$$U(x, \lambda) = T((x_0 + x_1)/2) V_m((x_1 - x_0)/2) \Delta^{-\mathrm{i}\frac{\lambda}{2\pi}} .$$

Show that this defines the mass m (spin=0) Wigner representation on 1+1D Minkowski space.
Note: Then H can identified with the standard space $H(W)$ associated with the wedge $W = \{x \in \mathbb{R}^{1+1}, |x_0| < x_1\}$. So we extended (H, T) which gives a dilation-translation covariant net on the light ray, to a net on 1+1D Minkowski space.

Exercise: CAR algebra

Let $\mathcal{H}$ be a Hilbert space and $\Lambda(\mathcal{H})$ be the Hilbert space with scalar product:

$$(e_1 \wedge \cdots \wedge e_n, f_1 \wedge \cdots \wedge f_m) := \delta_{m,n} \det((e_i, f_j)_{i,j}) .$$

Show that $a(f)\xi = f \wedge \xi$ defines a bounded operator on $\Lambda(H)$, and show that:

$$a(f)^* a(g) + a(g) a(f)^* = (f, g) \cdot 1$$

holds.